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# GENERALIZED BINARY VARIABLE APPROACH TO SOLVE MULTI-CHOICE TRANSPORTATION PROBLEM

**Jadunath Nayak* Sudarsan Nanda† Srikumar Acharya‡**

A step to make Transportation problem more applicable in real life situation is the use of multi-choice for each factors like cost coefficients, demands and supplies. In this paper binary variables are used to find out an equivalent model for the given transportation problem. And the result is compared with the result when Interpolating polynomial is used to find the equivalent model. The aim of the paper is to find out a proper way to choose appropriate binary variables for finding an equivalent model for given multi choice Transportation problem. An example illustrates the proposed method and the result was compared with existing method of using Interpolating Polynomial. LINGO is used to solve the problem.

Keywords: Multi-choice decision making, transportation problem. Binary Variables

## 1 Introduction

Transportation problem deals with the cost of transportation of various goods from sources to destinations where all values available are fixed numbers. But in real life situation the numbers are not crisp, they are either fuzzy in nature or involve several multi-choice factors.

After development of Fuzzy theory by Zadeh[14] many researchers used Fuzzy for making transportation problem more realistic and useful in real life situation.Shiang Tai Liu and Chiang Kao[9], Chanas et al. [5], Chanas and Kuchta[6] proposed methods for solving Fuzzy Transportation Problem of this type. Pandian et al. [10] proposed fuzzy zero point method for finding optimal methods for a fuzzy Transportation problem where parameters are trapezoidal fuzzy numbers. Gani et al.[7] obtained a fuzzy solution for a two stage cost minimizing fuzzy Transportation problem where supplies and demands are trapezoidal fuzzy numbers.Similarly Hadi Basir and Zadeh [2] gave a fuzzy approach to solve Transportation problem.

But since development of multi-choice by Healy[8]researchers got a new field to use Linear programming Problems in a real life situation. Biswal and Acharya[3], Acharya and Biswal[4]converted multi choice Linear Programming problem to an equivalent non-Linear Problem using Binary Variable and Interpolating polynomial.Roy et al.[13] and Roy[11] gave approaches to solve Multi- Choice Stochastic transportation Problem using standard Mathematical programming and Binary variable. Roy[12] used Lagrange's interpolating polynomial to solve a multi choice transportation problem where all the factors like cost co efficient, demands and availability are multi- choice in nature.Acharya and Acharya[1] generalized the step to follow to findout appropiate binary variable replacement for a multi-choice Linear programming problem (LPP).

In this paper we use binary variables to convert a Multi-Choice Transportation Problem (MCTP) to an equivalent model, and the result is compared with the result of such problems which are converted with Interpolating polynomial.

* e-mail-jadunath76@gmail.com, Baripada College, Baripada, Odisha
† e-mail-snanda@kiit.ac.in, KIIT University, Bhubaneswar, Odisha
‡ e-mail-sacharyafma@kiit.ac.in, KIIT University, Bhubaneswar, Odisha

### *2 Mathematical Model*

Consider a Transportation problem with m origin (sources) and n destination. Let $\{a_i^{(1)}, a_i^{(2)}, \ldots, a_i^{(K)}\}$ be the multiple choice availability at $i^{th}$ destination and $\{b_j^{(1)}, b_j^{(2)}, \ldots, b_j^{(K)}\}$ be the multi choice requirement at $j^{th}$ destination and let { $C_{ij}^{(1)}, C_{ij}^{(2)} \ldots, C_{ij}^{(k)}$} be multi choice cost co efficient for transporting a unit from $i^{th}$ source to $j^{th}$ destination.

then the problem is

$$\min : Z = \sum_{i=1}^{n}\sum_{j=1}^{n}\{C_{ij}^{(1)}, C_{ij}^{(2)} ..., C_{ij}^{(k)}\}X_{ij} \tag{1}$$

Subject to

$$\sum_{i=1}^{m} X_{ij} \le \{a_i^{(1)}, a_i^{(2)}, \ldots, a_i^{(K)}\} \tag{2}$$

$$\sum_{j=1}^{n} X_{ij} \ge \{b_j^{(1)}, b_j^{(2)}, \ldots, b_j^{(K)}\} \tag{3}$$

$$X_{ij} \ge 0 \tag{4}$$

As the problem cannot solve directly we find out an equivalent model where the multi choice factors are replace by some other factors. Roy[12]used Lagrange's interpolating polynomial and in our case we consider Binary variables.

### *3 Equivalent Model*

To form an equivalent model by using binary variable to make a suitable choice out of given multi choice we first choose required numbers of binary variables. This can be done using following steps

#### 3.1 Choice of Binary variables

**Step-1**

Find number of multi choice for each cost coefficient if cost of transportation from $i^{th}$ source to $j^{th}$ destination be $\{C_{ij}^{(1)}, C_{ij}^{(2)}, \ldots, C_{ij}^{(k_{ij})}\}$ Then total number of choices for $ij_{th}$ cost coefficient is $K_j$. We suppose that $K_j \ge 2$ ( choice is obvious if $K_j = 1$)

**Step-2**

Find the total number of binary variable required to handle the multi choice at $ij_{th}$ cost coefficient. This can be done in following manner.Find $l_{ij}$, for which $2^{(l_{ij}-1)} \le k_{ij} \le 2^{(l_{ij})}$. Hear $L_{ij}$ is number of binary variable required. Let $z_{ij}^1, z_{ij}^2, \ldots, z_{ij}^{l_{ij}}$ are the binary variables.

**Step-3**

Expand $2^{l_{ij}}$ as $\binom{l_{ij}}{1}+\binom{l_{ij}}{2}+\ldots+\binom{l_{ij}}{r_{i1}}+\ldots+\binom{l_{ij}}{r_{i2}}+\ldots+\binom{l_{ij}}{l_{ij}}$ and select the smallest number of consecutive terms whose sum is 'equal to' or 'just greater then' $k_{ij}$ from the expansion. Let these term are $\binom{l_{ij}}{r_{i1}},\binom{l_{ij}}{r_{i1+1}},\binom{l_{ij}}{r_{i1+2}}\ldots\binom{l_{ij}}{r_{i2}}$.

**Step-4**

Now convert each multi choice $\{X_{ij}Cij^{(1)}, X_{ij}Cij^{(2)},\ldots,X_{ij}Cij^{(k_{ij})}\}$ by using $k_{ij}$ binary code as follows

$$\{C_{ij}^{(1)},C_{ij}^{(2)},\ldots,C_{ij}^{(k_{ij})}\}X_{ij}=[\sum_{k=1}^{\binom{l_{ij}}{r_{i1}}}P_{ij}^{r_{i1}}Q_{ij}^{i1}C_{ij}^{(k)}+\sum_{k=1}^{\binom{l_{ij}}{r_{i1}+1}}P_{ij}^{r_{i1}+1}Q_{ij}^{i1+1}C_{ij}^{(\binom{l_{ij}}{r_{i1}}+k)}$$
$$+\ldots+\sum_{k=1}^{\binom{l_{ij}}{r_{i2}-1}}P_{ij}^{r_{i2}-1}Q_{ij}^{i2-1}C_{ij}^{(\binom{l_{ij}}{r_{i1}}+\ldots+\binom{l_{ij}}{r_{i2}-2}+(k))}$$
$$+\sum_{k=1}^{k_{ij}-L_{ij}^{(1)}}P_{ij}^{r_{i2}}Q_{ij}^{i2}C_{ij}^{L_{ij}^{(1)}+(k)}]X_{ij} \tag{5}$$

Where $L_{ij}^{(1)}=\binom{l_{ij}}{r_{i1}}+\binom{l_{ij}}{r_{i1+1}}+\ldots+\binom{l_{ij}}{r_{i2-1}}$

$$P_j^{(si)}=\{z_{ij}^{(j1)}.z_{ij}^{(j2)}\ldots.z_{ij}^{(js)}\,|\,\{j_1,j_2,\ldots,j_s\}\in I_s^{(j)}, S=r_{i1},r_{i1+1},\ldots,r_{i2}\}$$
$$Q_j^{(si)}=\{\textstyle\prod_{j=1}^{l_i}(1-z_{ij}^{(j)})\,|\,j\ni\{j_1,j_2,\ldots,j_s\}\}$$
$$I_s^{(j)}=\{\{j_1,j_2,\ldots,j_s\}\,|\,j_1<j_2<\ldots<j_s, s=r_{i1},r_{i1+1},\ldots r_{i2}\}$$
$$j_1\in\{1,2,\ldots,(l_i-s)+1\}, j_2\in\{2,3,\ldots(l_i-s)+2\},\ldots j_s\in\{s,s+1,\ldots,l_i\}$$

**Step-5**

To overcome repetition of $(2^{l_{ij}}-k_{ij})$ Binary codes we impulse the following restriction.

$$z_{ij}^{(1)}+z_{ij}^{(2)}+z_{ij}^{(3)}+\ldots+z_{ij}^{(l_i)}\geq r_{i1}$$
$$z_{ij}^{(1)}+z_{ij}^{(2)}+z_{ij}^{(3)}+\ldots+z_{ij}^{(l_i)}\leq r_{i2}$$
$$z_{ij}^{(j1)}+z_{ij}^{(j2)}+z_{ij}^{(j3)}+\ldots+z_{ij}^{(jr_{i2})}\leq r_{i1}$$
$$j=(k_i-L_{ij}^{(1)})+1,(k_i-L_{ij}^{(1)})+2,\ldots,\binom{l_{ij}}{r_{i2}}$$

**Step-6**

Now formulate the given multi choice programming problem as

$$Max:Z=\sum_{i=1}^{n}\sum_{j=1}^{m}[\sum_{k=1}^{\binom{l_i}{r_{i1}}}P_{ij}^{r_{i1}}Q_{ij}^{i1}C_{ij}^{(k)}+\sum_{k=1}^{\binom{l_i}{r_{i1}+1}}P_{ij}^{r_{i1}+1}Q_{ij}^{i1+1}C_{ij}^{(\binom{l_i}{r_{i1}}+k)} \tag{6}$$

$$+\ldots+\sum_{k=1}^{\binom{l_i}{r_{i2}-1}} P_{ij}^{r_{i2}-1} Q_{ij}^{i2-1} C_{ij}^{(\binom{l_i}{r_{i1}}+\ldots+\binom{l_i}{r_{i2}-2}+(k)} \quad (7)$$

$$+\sum_{k=1}^{k_{ij}-L_i^{(1)}} P_{ij}^{r_{i2}} Q_{ij}^{i2} C_{ij}^{L_i^{(1)}+(k)} ]X_{ij} \quad (8)$$

$$\sum_{i=1}^{m} X_{ij} \leq g(\alpha_i^k) \quad (9)$$

$$\sum_{j=1}^{n} X_{ij} \geq h(\beta_j^k) \quad (10)$$

$$X_{ij} \geq 0 \quad (11)$$

Where

$$z_{ij}^{(1)}+z_{ij}^{(2)}+z_{ij}^{(3)}+\ldots+z_{ij}^{(l_i)} \geq r_{i1}$$

$$z_{ij}^{(1)}+z_{ij}^{(2)}+z_{ij}^{(3)}+\ldots+z_{ij}^{(l_i)} \leq r_{i2}$$

$$z_{ij}^{(j1)}+z_{ij}^{(j2)}+z_{ij}^{(j3)}+\ldots+z_{ij}^{(jr_{i2})} \leq r_{i1}$$

$$j=(k_i-L_i^{(1)})+1,(k_i-L_i^{(1)})+2,\ldots,\binom{l_i}{r_{i2}}$$

And $g(\alpha_i^k), h(\beta_j^k)$ are similar choice of binary code as discuss.

**Step-7**

Above mathematical model is a non linear mixed integer programming problem. This model can be solved by using existing method like LINGO.

***4 Example***

A petroleum process crude oil to produce petroleum product. The company has three storage terminals and four depots. The company transports refined oil from storage terminal to depots via various ways like tankers and railways etc.The cost of transportation ( in rupees )for one unit of refined oil are treated as multi choice parameter due to various ways of transportation. Again due to manpower electricity and other factors the supply amount is multi-choice in nature. Due to unpredictable market condition the demand at various depots are also multi choice in nature. The table below shows all the factors.

| | $D_1$ | $D_2$ | $D_3$ | *Supply* |
|---|---|---|---|---|
| $S_1$ | {16,19} | {18,19} | {14,16} | {7,9,10 |
| $S_2$ | {20,21} | {13,16, | {14,17, | {10,12} |
| $S_3$ | {15,16} | {20,21} | {10,12} | {4,5} |
| *Deman* | {9,11} | {3,4,6} | {5,6,7} | |

### 4.1 Mathematical model of the problem

mathematically the above problem can be stated as.

$$\begin{aligned}\min : Z &= \{16,19\}X_{11}+\{18,19\}X_{12}+\{14,16\}X_{13}\\ &+\{20,21\}X_{21}+\{13,16,18\}X_{22}+\{14,17,19,22\}X_{23}\\ &+\{15,16\}X_{31}+\{20,21\}X_{32}+\{10,12\}X_{33}\end{aligned} \quad (12)$$

Subject to

$$\sum_{j=1}^{3} X_{1j} \le a_1, a_1 \in \{7,9,10\} \quad (13)$$

$$\sum_{j=1}^{3} X_{2j} \le a_2, a_2 \in \{10,12\} \quad (14)$$

$$\sum_{j=1}^{3} X_{3j} \le a_3, a_3 \in \{4,5\} \quad (15)$$

$$\sum_{i=1}^{3} X_{i1} \ge b_1, b_1 \in \{9,11\} \quad (16)$$

$$\sum_{i=1}^{3} X_{i2} \ge b_2, b_2 \in \{3,4,6\} \quad (17)$$

$$\sum_{i=1}^{3} X_{i3} \ge b_3, b_3 \in \{5,6,7\} \quad (18)$$

$$X_{ij} \ge 0, \forall i \,\&\, j \quad (19)$$

### 4.2 Solution using Binary variables Approach

For clarify the proposed method of choosing appropriate binary code we consider the case at (2,3). there are four multi choice cost coefficient. i.e $k_{23} = 4$

Now $2^1 < k_{23} \le 2^2$ thus $l_{ij} = 2$

Thus we need 2 binary variables say $\theta_{23}^{(1)}, \theta_{23}^{(2)}$

Now $2^2 = \binom{2}{0}+\binom{2}{1}+\binom{2}{2}$ hence

$\{14,17,19,22\}X_{23}$

$= [[\theta_{23}^{(1)}.\theta_{23}^{(2)}]14+[\theta_{23}^{(1)}.(1-\theta_{23}^{(2)})]17+[(1-\theta_{23}^{(1)})\theta_{23}^{(2)}]19+[(1-\theta_{23}^{(1)})(1-\theta_{23}^{(2)})]22]X_{23}$

By using similar techniques we can convert all multi choice factors as.

$$\{16,19\}X_{11} = (16\theta_{11}+19(1-\theta_{11}))X_{11} \quad (20)$$

$$\{18,19\}X_{12} = (18\theta_{12}+19(1-\theta_{12}))X_{12} \quad (21)$$

$$\{14,16\}X_{13} = (14\theta_{13}+16(1-\theta_{13}))X_{13} \quad (22)$$

$$\{20,21\}X_{21} = (20\theta_{21}+21(1-\theta_{21}))X_{21} \quad (23)$$

$$\{13,16,18\}X_{22} = (13\theta_{22}^{(1)}\theta_{22}^{(2)}+16\theta_{22}^{(1)}(1-\theta_{22}^{(2)})+18(1-\theta_{22}^{(1)})\theta_{22}^{(2)})X_{22} \quad (24)$$

$$\{14,17,19,22\}X_{23} = (14(\theta_{23}^{(1)}.\theta_{23}^{(2)})+17(\theta_{23}^{(1)}.(1-\theta_{23}^{(2)})) + 19((1-\theta_{23}^{(1)})\theta_{23}^{(2)})+22((1-\theta_{23}^{(1)})(1-\theta_{23}^{(2)}))X_{23} \quad (25)$$

$$\{15,16\}X_{31} = (15\theta_{31}+16(1-\theta_{31}))X_{31} \quad (26)$$

$$\{20,21\}X_{32} = (20\theta_{32}+21(1-\theta_{32}))X_{32} \quad (27)$$

$$\{10,12\}X_{33} = (10\theta_{33}+12(1-\theta_{33}))X_{33} \quad (28)$$

Similarly demand and restriction can be converted using binary variables.

Now the above conversion we can state the multi choice transportation problem to a mixed integer non-linear programming problem which state as.

$$\begin{aligned}\min : Z &= (16\Theta_{11}+19(1-\Theta_{11}))X_{11}+(18\Theta_{12}+19(1-\Theta_{12}))X_{12}+(14\Theta_{13}+16(1-\Theta_{13}))X_{13}\\ &+(20\Theta_{21}+21(1-\Theta_{21}))X_{21}+(13\Theta_{22}^{1}\Theta_{22}^{2}+16\Theta_{22}^{1}(1-\Theta_{22}^{2})+18(1-\Theta_{22}^{1})\Theta_{22}^{2})X_{22}\\ &+[[\theta_{23}^{(1)}.\theta_{23}^{(2)}]14+[\theta_{23}^{(1)}.(1-\theta_{23}^{(2)})]17+[(1-\theta_{23}^{(1)})\theta_{23}^{(2)}]19+[(1-\theta_{23}^{(1)})(1-\theta_{23}^{(2)})]22]X_{23}\\ &+(15\Theta_{31}+16(1-\Theta_{31}))X_{31}+(20\Theta_{32}+21(1-\Theta_{32}))X_{32}+(10\Theta_{33}+12(1-\Theta_{33}))X_{33}\end{aligned} \quad (29)$$

Subject to

$$\sum_{j=1}^{3} X_{1j} \le [7\alpha_1^1\alpha_1^2+9\alpha_1^1(1-\alpha_1^2)+10(1-\alpha_1^1)\alpha_1^2 \quad (30)$$

$$\sum_{j=1}^{3} X_{2j} \le [10\alpha_2+12(1-\alpha_2] \quad (31)$$

$$\sum_{j=1}^{3} X_{3j} \le [4\alpha_3+5(1-\alpha_3)] \quad (32)$$

$$\sum_{i=1}^{3} X_{i1} \ge [9\beta_1+11(1-\beta_1)] \quad (33)$$

$$\sum_{i=1}^{3} X_{i2} \ge [3\beta_2^1\beta_2^2+4\beta_2^1(1-\beta_2^2)+6(1-\beta_2^1)\beta_2^2] \quad (34)$$

$$\sum_{i=1}^{3} X_{i3} \ge [5\beta_3^1\beta_3^2+6\beta_3^1(1-\beta_3^2)+7(1-\beta_3^1)\beta_3^2] \quad (35)$$

$$X_{ij} \ge 0, \forall\, i \,\&\, j \quad (36)$$

$$\Theta_{11},\Theta_{12},\Theta_{13},\Theta_{21},\Theta_{22}^{(1)},\Theta_{22}^{(2)},\Theta_{23}^{(1)},\Theta_{23}^{(2)},\Theta_{31},\Theta_{32},\Theta_{33} \in \{0,1\}$$

$$\alpha_1^1,\alpha_1^2,\alpha_2,\alpha_3 \in \{0,1\}$$

$$\beta_1,\beta_2^1,\beta_2^2,\beta_3 \in \{0,1\}$$

This problem is solved using LINGO 15 and the solution is $X_{11}=4, X_{12}=0, X_{13}=0, X_{21}=0, X_{22}=0, X_{23}=8, X_{31}=5, X_{32}=0, X_{33}=0$ and minimum value of Z is 139.

### 4.3 Solution Using Interpolating Polynomial Approach

Using Lagrange's Interpolating Polynomial the above problem can be stated as

$$\min: Z = (3Z_{11}+16)X_{11}+(Z_{12}+18)X_{12}+(2Z_{13}+14)X_{13}+(Z_{21}+20)X_{21} + (-\frac{1}{2}Z_{22}^2+\frac{7}{2}Z_{22}+13)X_{22}+(\frac{1}{2}Z_{23}^3-\frac{5}{2}Z_{23}^2+6Z_{23}+13)X_{23} + (Z_{31}+15)X_{31}+(Z_{32}+20)X_{32}+(2Z_{33}+10)X_{33} \quad (37)$$

Subject to

$$\sum_{j=1}^{3} X_{1j} \le [-\frac{1}{2}Y_1^2+\frac{5}{2}Y_1+7] \quad (38)$$

$$\sum_{j=1}^{3} X_{2j} \le [2Y_2+10] \quad (39)$$

$$\sum_{j=1}^{3} X_{3j} \le [Y_3+4] \quad (40)$$

$$\sum_{i=1}^{3} X_{i1} \ge [2Z_1+9] \quad (41)$$

$$\sum_{i=1}^{3} X_{i2} \ge [\frac{1}{2}Z_2^2+\frac{1}{2}Z_2+3] \quad (42)$$

$$\sum_{i=1}^{3} X_{i3} \ge [Z_3+5] \quad (43)$$

$$X_{ij} \ge 0, \forall\, i \,\&\, j \quad (44)$$

$$\{Z_{11}, Z_{12}, Z_{13}, Z_{21}, Z_{31}, Z_{32}, Z_{33}, Y_2, Y_3, Z_1\} \in \{0,1\}$$

$$\{Z_{22}, Z_{23}, Y_1, Z_2, Z_3\} \in \{0,1,2\}$$

This problem was solve using LINGO 15 and the result is $X_{11}=1, X_{12}=0, X_{13}=1, X_{21}=3, X_{22}=0, X_{23}=0, X_{31}=5, X_{32}=0, X_{33}=0$ Minimum value of transportation is 151 rupees.

### *5 Result discussion and conclusion*

In this paper we considered a Transportation Problem in which all the cost coefficients, sources and demands are multi-choice in nature.The equivalent non-linear model was formed using Binary variables. In the numerical example we solved the problem using both Interpolating Polynomial approach and Binary variables approach. The result shows that Binary variable approach is more appropriate than Interpolating polynomial approach. But in case of Binary Variable approach there are more non-linear variables then Interpolating Polynomial approach. Further research and study can clarify which was more appropriate for application.

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